Lecture Notes on Basic Set Theory

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1 Introduction

Set theory is one of the foundations of modern mathematics. It is a branch of mathematics that deals with the study of sets, which are collections of objects. In this lecture, we have covered the basics of set theory, including set notation, operations on sets, and set relations. We have also introduce function in terms of relation.

2 Set Notation

A **set** is a well-defined collection of distinct objects which is in our thought or intuition. We denote a set by enclosing its elements in curly braces {}, for example:

$$
\begin{aligned}
A &= \{1, 2, 3, 4\} \\
B &= \{a, b, c\} \\
C &= \{x \in \mathbb{R} : x > 0\}
\end{aligned}
$$

We use the symbol $\in$ to indicate that an element belongs to a set, and $\notin$ to indicate that an element does not belong to a set. For example, $1 \in A$ and $5 \notin A$.

3 Operations on Sets

There are several operations that can be performed on sets.

3.1 Union and Intersection

The **union** of two sets A and B, denoted by $A \cup B$, is the set of elements that belong to either A or B, or both:

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

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The **intersection** of two sets A and B, denoted by $A \cap B$, is the set of elements that belong to both A and B:

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Example 3.1. Suppose we have two sets $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$.

Then the union of A and B, denoted by $A \cup B$, is the set of all elements that are in A or B, or both. Thus, we have:

$$A \cup B = \{1, 2, 3\} \cup \{2, 3, 4\} = \{1, 2, 3, 4\}$$

Similarly, the intersection of A and B, denoted by $A \cap B$, is the set of all elements that are in both A and B. Thus, we have:

$$A \cap B = \{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\}$$

3.2 Set Difference

The **set difference** of two sets A and B, denoted by $A \setminus B$, is the set of elements that belong to A but not to B:

$$A \setminus B = \{x : x \in A \text{ and } x \notin B\}$$

Example 3.2. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 4, 5, 6, 7\}$. Then, the set difference $A \setminus B$ is:

$$A \setminus B = \{1, 2\}$$

Similarly, the set difference $B \setminus A$ is:

$$B \setminus A = \{6, 7\}$$

3.3 Cartesian Product

The **Cartesian product** of two sets A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$:

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

Example 3.3. Let $A = \{1, 2\}$ and $B = \{a, b, c\}$. Then, the cartesian product of A and B, denoted by $A \times B$, is:

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$$

Here, each element of A is paired with each element of B to form a new element in the Cartesian product. The resulting set contains all possible ordered pairs of elements from A and B.

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4 Set Relations

A **relation** between two sets A and B is a subset of the Cartesian product $A \times B$. If (a, b) belongs to the relation, we say that a is related to b.

Example 4.1. Suppose we have two sets $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. We can define a relation R between elements of A and B as follows:

$$R = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$$

This relation R relates element 1 of set A to element 2 and 3 of set B, element 2 of set A to element 2 and 3 of set B, and element 3 of set A to element 3 of set B. We can write this relation in set-builder notation as:

$$R = \{(a, b) : a \in A, b \in B, a = b \text{ or } a < b\}$$

4.1 Functions

A **function** f from a set A to a set B, denoted by $f : A \to B$, is a relation that assigns each element $a \in A$ to a unique element $b \in B$.

Example 4.2. Consider the set $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. Let f be a relation from A to B defined by $f = \{(1, 4), (2, 5), (3, 6)\}$.

To show that f is a function, we need to verify that for each $a \in A$, there exists a unique $b \in B$ such that $(a, b) \in f$.

We can see that for $a = 1$, $(a, b) = (1, 4) \in f$. Similarly, for $a = 2$, $(a, b) = (2, 5) \in f$, and for $a = 3$, $(a, b) = (3, 6) \in f$.

Thus, for each $a \in A$, there exists a unique $b \in B$ such that $(a, b) \in f$, and therefore f is a function.

4.2 Try Exercises

Determine whether the following relation is a function or not:

1. $R_1 = \{(1, 2), (3, 4), (1, 5), (6, 2)\}$
2. $R_2 = \{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4)\}$
3. $R_3 = \{(1, 2), (3, 4), (5, 6), (7, 8)\}$
4. $R_4 = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$
5. $R_5 = \{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10)\}$
6. $R_6 = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (6, 5)\}$
7. $R_7 = \{(1, 2), (3, 4), (5, 6), (7, 8)\}$
8. $R_8 = \left\{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 1\right\}$
9. $R_9 = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x + y = 1\}$.

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